INVESTIGATING THE STUDENTS' UNDERSTANDING OF CURVE PROPERTY-RELATED DERIVATIVE PROBLEMS USING A TEACHING EXPERIMENT

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ABSTRACT

Traditional methods of teaching differentiation calculus usually involve lectures. Nevertheless, this process impedes students' thorough comprehension of concepts and their practical application by promoting dependence on memorisation and procedural learning. Numerous studies have also denoted that teaching experiments and technology can enhance mathematics learning concerning differentiation topics. Therefore, this study proposed a teaching experiment to examine the pre-university students' understanding of curve property-related derivative problems. The proposed strategy grounded in mathematics-related education presented two primary outcomes: (a) data regarding student understanding and (b) adoption of student-centred instruction. This strategy also involved the Action, Process, Object, and Schema (APOS) theory in assessing the interview and worksheet-based data. Consequently, the students held two misconceptions about positive derivative functions: (a) the positive derivative functions were erroneously correlated with an increasing interval on the derivative function, and (b) misinterpretation of negative value derivative functions as decreasing intervals on the derivative functions. These students also encountered difficulties in aligning cubic graphs with their corresponding derivatives. The instructional practices for the subsequent cycle were then improved based on the results of this analysis. Overall, improved mathematical understanding and practical instructional strategies could be realised by employing teaching experiments as an effective strategy to address these issues.

Keywords: *APOS theory, Desmos platform, differential calculus, teaching experiment.*

INTRODUCTION

The evolving demands of the contemporary world have driven substantial change in mathematics-related education in recent years, emphasising the necessity of technology in classrooms. This transition has prompted the emergence of technology-based curricula in global educational systems in facilitating graphing tools, interactive exploration of mathematical concepts, and computer-based problem-solving practices (Erens & Eichler, 2018).

Malaysia has incorporated differentiation topics into its additional mathematics syllabus as a mathematical skill that develops alongside calculus for secondary schools and pre-university levels (Hamda et al., 2020). These topics are significant across multiple disciplines, requiring students to establish a robust foundation in this area. Several fields critically employing these topics include physics, engineering, economics, and computer science. Notably, differentiation topics represent an entry point for advanced mathematical concepts by enhancing overall calculus competence. Although these topics have been highly acknowledged, students still frequently encounter difficulties in understanding calculus (Jones & Watson, 2018).

A pressing issue in current mathematics education is the persistent gap between procedural proficiency and conceptual understanding of calculus, particularly in differentiation. Despite students being able to perform derivative calculations, they often fail to interpret these results in context, especially when analysing the properties of curves such as concavity. Recent studies showcase that even when using technology tools such as GeoGebra, students significantly improve in conceptual understanding but exhibit no equivalent gains in procedural fluency (Bedada & Machaba, 2022). Moreover, an investigation into the reciprocal relationship between procedural and conceptual knowledge found that students employ procedural methods frequently without fully comprehending underlying concepts (Hechter et al., 2022). In Malaysia, this dynamic is especially concerning. Despite students demonstrating competence in differentiation procedures, they exhibit poor conceptual engagement. This suggested that traditional lecture-based instruction continues to reinforce memorisation over deep understanding (Yatim et al., 2022).

The urgency of addressing this problem is highlighted by the increasing evidence that technology-enhanced, student-centred instruction can significantly improve conceptual understanding in mathematics (Bukhatwa et al., 2022). However, in the Malaysian context, the application of such approaches remains limited, with few empirically grounded studies examining how students interact with derivative concepts through graph-based technology tools with constructivist learning frameworks. This study responds to this gap by designing a teaching experiment based on the Action, Process, Object, and Schema (APOS) theory, aiming to reveal students' reasoning processes and address their misconceptions about curve-related derivative properties. By doing so, the study not only provides timely insights into how students think about calculus concepts through APOS theory but also contributes a novel instructional model grounded in local education and students' needs.

Despite innovative teaching techniques often promoted by the Malaysian Qualifications Agency (MQA), numerous educators lack clear guidelines and technology-integrated teaching aids (Malaysian Qualifications Agency, 2019). This process can impede effective student engagement and conceptual understanding. Steffe and Thompson (2000) proposed one example to resolve this issue, which employed student-centred and constructivist instruction through innovative strategies. The study introduced Realistic Mathematics Education (RME) to shift the focus from rote memorisation to developing conceptual understanding and problem-solving abilities.

Mathematics-related education often applies teaching experiments as a research strategy to investigate student comprehension while developing effective instructional practices (Geiger et al., 2015). Instructors can then assist in assessing students' conceptual understanding and enhancing problem-solving skills by presenting mathematical concepts within context in encouraging students to explore, reason, and construct their knowledge (Hajra, 2013). Given that teaching experiments have been infrequently examined in Malaysian mathematics-related education studies, this study established a teaching experiment to enhance pre-university students' understanding of curve property-related derivative problems. The objectives of this study are as follows:

1. To assess the students' understanding of derivatives using the Action, Process, Object, and Schema (APOS) theory.
2. To examine the challenges and misunderstandings encountered by students in solving curve property-related derivative problems.

Students' Understanding of Calculus

The APOS (Actions, Processes, Objects, and Schemas) theory identifies essential elements in cognitive development during students' engagement with mathematical concepts, advancing from tangible actions to abstract cognitive frameworks (Arnon et al., 2014). Even though students initially participate in physical or mental activities associated with mathematical concepts (basic operations) in the Action

(A) stage, they may not possess a thorough understanding (Dubinsky & McDonald, 2001). The Process (P) stage then involves the students concentrating on mental operations and reasoning to advance abstract thinking-related tasks such as comparison and generalisation (Listiawati & Juniati, 2021).

The students focus on the mathematical objects in the Object (O) stage, reflecting a more profound comprehension. Maharaj (2013) denoted that this stage occurred if individuals became aware of its entirety and could construct or apply transformations in various contexts. The Schema (S) stage entails the integration of mathematical objects and processes into coherent mental frameworks. This process indicates a higher level of comprehension, in which students acquire adaptable and interrelated frameworks.

Generally, science and engineering students at the university level encounter calculus and its derivatives as fundamental topics. Several studies have also employed theoretical frameworks to examine the learning and teaching of the derivative while understanding students' comprehension of the concepts (Huang, 2011; Roorda et al., 2007). Ozaltun-Celik (2021) explored the relationship between students' cognitive processes and the graphical interpretation of the derivative. The study discovered that students had difficulties relating the global view of derivatives to the concept of a function. Jones (2017) assessed the application of derivatives in non-kinematics contexts regarding students' comprehension. The study identified six elements of students' approaches to constructing understanding through students' interviews and surveys. Haghjoo and Reyhani (2021) documented frequent misconceptions concerning derivative concepts among undergraduates in basic sciences and engineering students. These studies emphasised the need for improved teaching techniques and a more profound comprehension of students' cognitive processes in learning derivative calculus. Therefore, this study assessed the students' understanding of derivative calculus based on APOS.

Numerous studies have successfully examined technological integration in calculus education and its effects on student achievement and attitudes (Albalawi, 2018; Ebert, 2015). Nevertheless, limited studies explore students' understanding of calculus through APOS within a technological learning environment in the context of Desmos. Thus, this study introduced an instructional lesson to evaluate students' comprehension of differential calculus based on APOS in a Desmos learning setting in addressing the research gap.

METHODOLOGY

Studies should examine the impact of the designed instructional lesson on student comprehension concerning differential calculus and the potential for enhancement through technology. Specifically, educators necessitate a flexible research approach, such as teaching experiments, to address the diversity of learning processes. Steffe and Thompson (2000) pioneered teaching experiments, while Bakker and van Eerde (2015) later adapted them for design-based research. This modification effectively linked research with teaching and learning processes while promoting the evolution of instructional lesson development (Bakker, 2004). The following subsections explain these teaching experiments and the methodology employed in this study.

Teaching Experiment

Teaching experiments serve as a research methodology in mathematics-related education to understand the students' mathematical learning and reasoning processes. This process presents two main objectives: (a) directly engaging with students' mathematical constructions and (b) assigning independent mathematical realities (Steffe & Thompson, 2000). Teaching experiments are usually performed in bolstering the initial instructional design through ongoing testing and revision of conjectures. Previous studies also demonstrated that assessing students' reasoning and the learning environment could accomplish these outcomes (Cobb et al., 2003). Hence, continuous analysis remains critical in obtaining the correlation data between student engagement and materials to modify instructional strategies to produce effective and customised learning experiences.

The teacher-researcher's formulation and examination of hypotheses in teaching episodes provide students with an adequate understanding of mathematical concepts. Considering that mathematical discourse is promoted through a supportive classroom environment, students can express and justify their mathematical concepts (Steffe, 1991). The procedure also involves student interviews with probing queries to uncover their reasoning and thought processes regarding the activities they are instructed to elucidate (Engelhardt et al., 2004). These student's conceptual understanding and mathematical reasoning processes obtained from teaching experiments are pivotal in questioning traditional assessment methods (Steffe & Thompson, 2000). Consequently, the student's comprehensive understanding of mathematics is focused rather than remaining on procedural accuracy.

Teaching experiments offer advantages over traditional clinical interviews by facilitating the evaluation of novel instructional techniques (Cobb et al., 2003; Wang & Hannafin, 2005). Instructors can then develop suitable curricular interventions while establishing the appropriate sequence of activities. Conceptual understanding and problem-solving abilities are also often observed in a student-centred, constructivist methodology based on teaching experiments. Nonetheless, derivative calculus lessons incorporating teaching experiments, APOS, and the Desmos platform have rarely been observed in Malaysia. Thus, this study applied these components to evaluate the students' understanding of calculus.

Participants and Lesson Description

This study conducted a teaching experiment in a science stream pre-university class at a private university in Sarawak, Malaysia. Nine students with diverse academic achievement levels participated in the assessment, including observations and interviews. This study also consisted of two teaching experiment cycles based on two elements: (a) activities-classroom discussion-exercise (ACE) (Asiala et al., 1997) and (b) the Desmos platform. Each cycle lasted four weeks with two-hour sessions weekly. Furthermore, a hypothetical learning trajectory was employed based on differential calculus instructional lessons (DCIL) evaluated through APOS (Simon, 1995). Each session of the teaching experiment then included the teacher-researcher, students, and a teaching witness, with collaborative modifications implemented following every episode (Molina et al., 2007). Nevertheless, this study specifically focused on the outcomes obtained from individual tasks and task-based interviews with nine students during the first cycle of the two teaching experiment cycles.

Data Collection

The data collection for the teaching experiment included analysing students' work and interviews. These task-based interviews utilised instruments from two sections (see Sections 2.3.1 and 2.3.3). The interviews were grounded in the genetic decomposition of the derivative calculus concept to examine the students' conceptual understanding, problem-solving approaches, and cognitive processes (Asiala et al., 1997; Charters, 2003). All obtained interview data were recorded and transcribed promptly following the first cycle completion of the teaching experiment. A meaningful comparison was ensured through a standardised questioning interview format to identify misconceptions and gaps in the student's mathematical understanding of instructional practices (Lebar, 2018; Maher & Sigley, 2014). Finally, a purposive sampling approach based on performance criteria was applied to select nine participants with different achievement levels (Rashidi et al., 2014). These students were consistently encouraged to express their thoughts and reasoning regarding the derivative task and instructional unit reflections.

This study did not involve the use of statistical or numerical data. The primary sources of data were qualitative in nature, consisting of students' work and interview outcomes. To ensure the reliability of the findings, data collection followed a structured process that included the use of a consistent interview protocol and the prompt transcription of all interview sessions. Reliability was further supported through the triangulation of multiple data sources, allowing for cross-verification of student responses (Creswell & Poth, 2018). As part of the validation process, transcripts were returned to the participating students for review to confirm the accuracy of their statements, a technique known as member checking, which helps enhance credibility and reduce researcher bias (Thomas, 2016). Additionally, the purposive sampling method, per the performance criteria, ensured the inclusion of a diverse range of student abilities, contributing to the overall trustworthiness and consistency of the collected data (Patton, 2015).

Interview Task 1: Derivative Task Involving Curve Properties. The $f'(x)$ graph was provided to the students in this derivative task. Meanwhile, the quadratic function $f'(x)$ graph demonstrated concave downward characteristics and a maximum point at $x = a$. These students were then tasked with identifying the value signs of $f'(x)$ and $f''(x)$ at designated points for part (a) of the question. Subsequently, these students were instructed to sketch the $f(x)$ graph in part (b).

Genetic Decomposition of Interview Task 1. This study conducted a genetic decomposition to apply APOS in analysing specific constructions created by students. The genetic decomposition functioned as a hypothetical learning trajectory that outlined the cognitive frameworks and processes necessary for students to develop and comprehend a specific mathematical concept (Arnon et al., 2014, p.27). Hence, the genetic decomposition for Task 1 is as follows:

Stage 1: Action. This stage involved the students determining the key features of the $f'(x)$ graph. The features were the positional relationship to the *x*-axis (whether above or below the *x*-axis) and accurate determination of the value sign for $f'(x)$.

Stage 2: Process. This stage involved the students analysing the graphs, recognising that derivatives produced quadratic shapes as slopes and trends (positive for increase, negative for decrease).

Stage 3: Object. This stage involved the students summarising the process by examining derivative characteristics. The procedure included analysing the shape, determining the position of the *x*-intercept based on stationary point coordinates, and identifying the presence of stationary points in the derivative graph.

Stage 4: Schema. This stage involved the students solidifying their comprehension by developing a mental schema representation. The students were expected to provide similarities and/or differences regarding shape, concavity, and stationary points for both derivative functions.

Interview Task 2: Cubic Graph Comparison. Figure 1 depicts the two cubic graphs on the Desmos platform presented to the students. The assignment involved students comparing the derivative graphs of two specified cubic graphs. These students were expected to assess the properties of the original cubic graphs and their corresponding derivatives using the derivative approach.

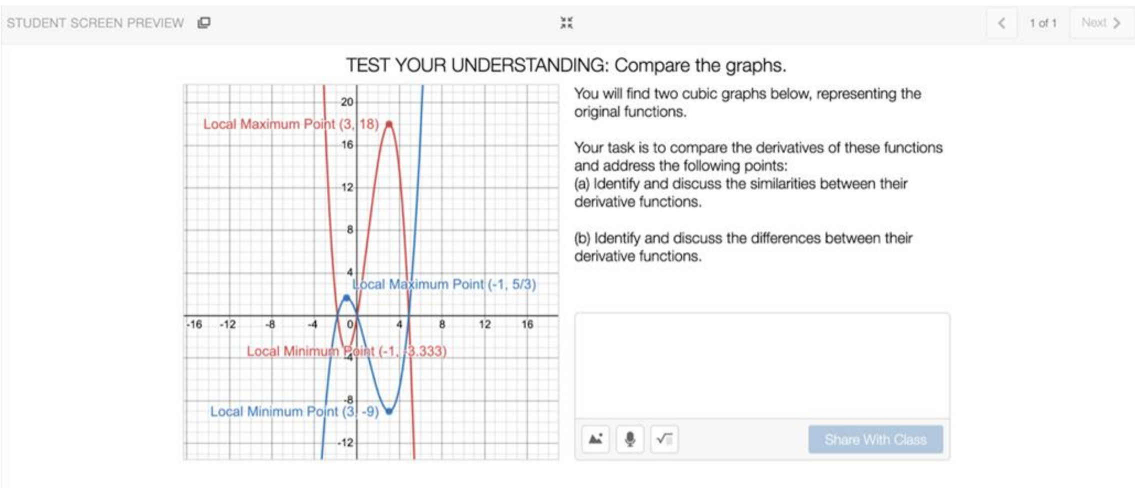

Figure 1. *The Questions Involved in Interview Task 2*

The APOS theory emphasises the progression of mathematical understanding. Therefore, the genetic decomposition for Task 2 is as follows:

Stage 1: Action. This stage involved the students determining the key features of both cubic graphs. The features were the positional relationship of the graph relative to the *x*-axis (whether above or below the *x*-axis) and the coordinates of the stationary points.

Stage 2: Process. This stage involved the students recognising that the derivative graph for a cubic function exhibited a quadratic form established through the differentiation process. The students were also expected to understand that the derivative of a function indicated the slope at any specific point. Thus, the derivative graph exhibited positive values when the function increased and vice versa.

Stage 3: Object. This stage involved the students encapsulating the process into an object conception, prompting them to analyse the characteristics of each derivative function. The analysis focused on the overall shape and the location of the *x*-intercept, which was determined by the *x*-coordinate of the stationary points from the original graph.

Stage 4: Schema. At this stage, the students can identify similarities and differences in the overall shape, concavity, and presence of stationary points in both derivative functions.

FINDINGS

Overall Interview Task 1 Results: Derivative Task Involving Curve Properties

Table 1 tabulates the overall classroom performance obtained from Interview Task 1, in which approximately 75% of the above-average students attain the Action stage. On the contrary, 50% of the above-average students achieved the Process stage based on the genetic decomposition utilising APOS. Approximately 75% of the above-average students also correctly identified the value sign of $f'(x)$ by analysing graph features concerning the positional relationship of the graph relative to the *x*-axis (whether above or below the *x*-axis). These Process stage achievers established the sign of $f''(x)$ through the analysis of derivative properties. Interestingly, only one student provided a partial sketch of the graph within the domain of $a \leq x \leq b$ (see Figure 2). Certain studies also demonstrated insufficient understanding in constructing a cubic graph from the original quadratic derivative graph, leading to erroneous representations.

Table 1. *Summary of Students' Results on The Derivative Task Involving Curve Properties*

Students' Performance	Action (%)	Process (%)	Object (%)	Schema (%)
Above-average	75	50	0	0
Average	80	40	0	0
Below-average	42.9%	0	0	0

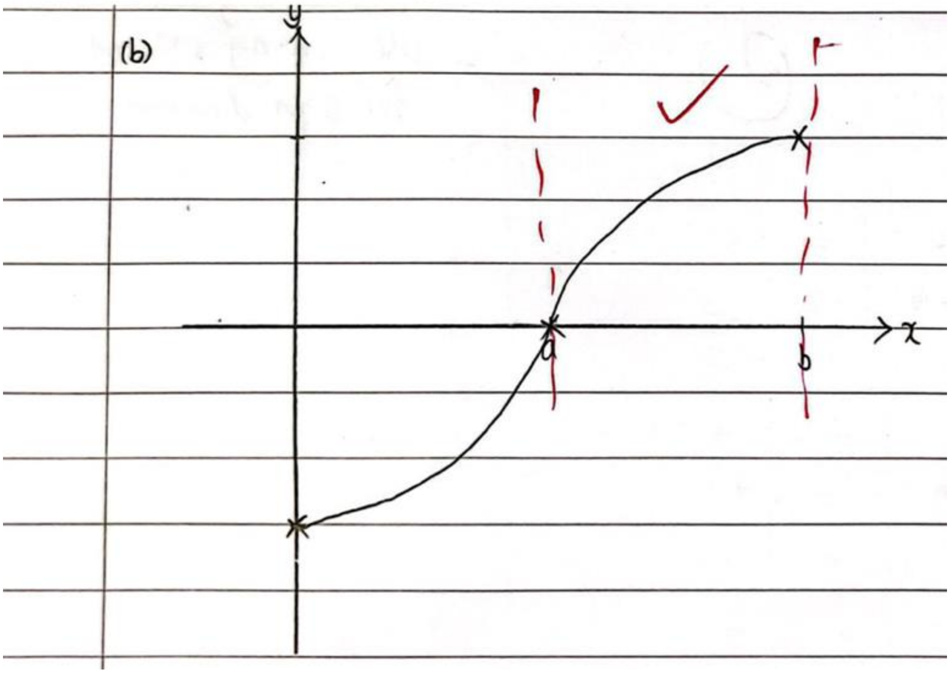

Figure 2. *A Working Sample for An Above-Average Student in Sketching the Graph of* $f(x)$

The average category revealed that 80% of students attained the Action stage, while 40% achieved the Process stage. Most participants in the below-average category also demonstrated inadequate proficiency, with only 42.9% reaching the Action stage in accurately determining the value sign for $f'(x)$. Conversely, no one accurately identified the value sign for $f''(x)$ or provided a sketch of it. Figure 3 displays a misconception from a below-average student. Even though this student depicted a quadratic graph as the derivative, it was orientated in a concave upward direction.

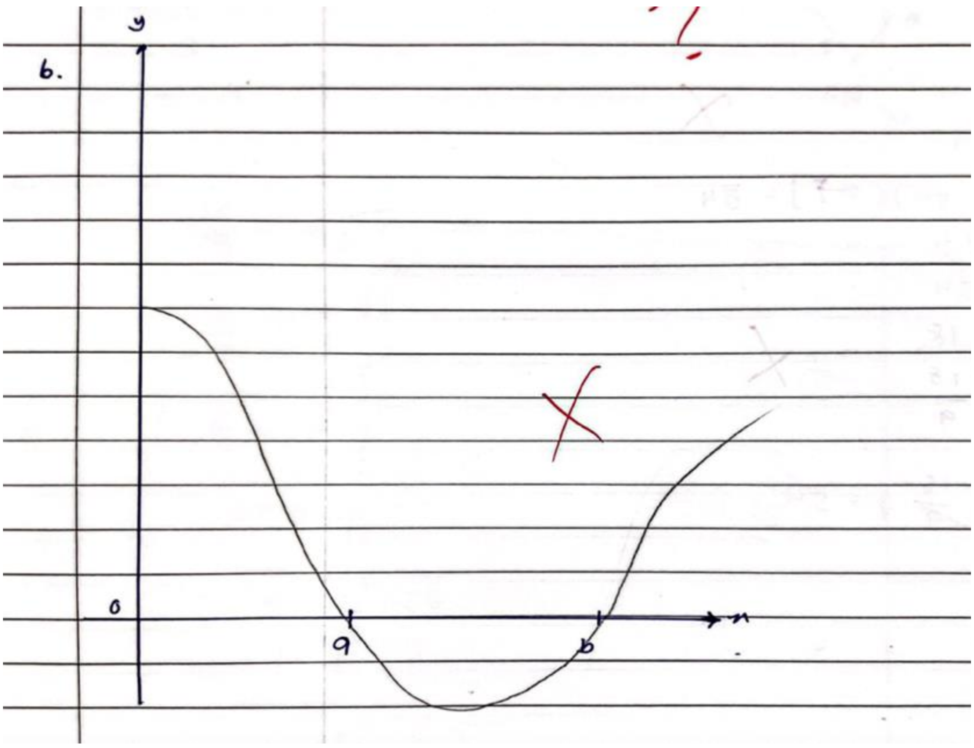

Figure 3. *The Working Sample for a Below-Average Student in Sketching the Graph of f(x)*

This study suggested that assuming students' understanding solely on a correct indication of value signs in part (a) should be avoided. The correct answers did not always reflect comprehensive knowledge, and incorrect answers did not inherently suggest misconceptions. Certain students also intentionally offered answers to obtain partial credit, which did not necessarily indicate genuine comprehension. Therefore, additional interviews were conducted with nine participants to gain students' understanding of derivatives and their approach to the task.

Task-Based Interview Outcomes

Nine students participated in individual task-based interviews following the lessons. Each category was represented by selecting three students: (a) above-average (S1, S2, and S3), (b) average (S4, S5, and S6), and (c) below-average (S7, S8, and S9). These interviews evaluated the students' understanding of derivative concepts, challenges encountered, and prevalent misconceptions.

Interview Outcomes for Below-Average Category Students

Understanding of Derivative Concepts. The students in the below-average category demonstrated the ability to reach the Action stage, signifying a fundamental understanding of the topic. Nonetheless, the students' comprehension could be confined to routine tasks and might not encompass more complex problem-solving situations. Only S8 could ascertain the value sign based on the position of a point. An excerpt from the interview with S8 is as follows:

Instructor	: How do we determine the value sign for the $f'(x)$?
S8	: I see the point in there.
Instructor	: Okay, when x is equal to a , what's the value sign for $f'(x)$?
S8	: Positive.
Instructor	: How about when x is equal to b?
S8	: Zero.
Instructor	: Why?
S8	: It's at x-axis.

(Interview S8: 36-43)

S7 accurately noted that the derivative graphs intersected at the same *x*-locations as the stationary points of the original function. Nevertheless, this student could not recognise that these points corresponded to the *x*-intercepts. An excerpt from the interview with S7 is as follows:

Instructor	: Maybe I will guide you on this. Did you notice that the blue colour graph has a local maximum at $x = -1$?
S7	: Yes.
Instructor	: Then, for the red colour graph it has a local minimum point. So, do you think there are any similarities in their derivatives?
S7	: They meet at the same point $x = -1$.
Instructor	: Just now, you say they will meet at the same point right? They will meet at where?
S7	: x equal to maximum so…. It will be here? (Pointed to maximum point) Or maybe not?

(Interview S7: 58-61;65-66)

S8 asserted that the derivative graphs intersected at the same *x*-locations as the stationary points of the original function, which aligned to the *x*-intercepts. Conversely, this student was conjecturing the answer without genuine comprehension. An excerpt from the interview with S8 is as follows:

S8 : The x-axis is the same.
Instructor : Okay, the x-intercept is the same for both graphs? Why do you say so?
S8 : I guess so.

(Interview S8: 67-69)

This understanding reflected a restricted comprehension of the connection between stationary points and the characteristics of the derivative function. Nonetheless, this comprehension varied among students, indicating differences in their understanding of the concept. Thus, additional investigation and reinforcement of this relationship were required to enhance their comprehension.

Task Difficulty. The students encountered various challenges in completing the assigned tasks, potentially affecting their performance. Notably, S9 encountered issues in ascertaining the value sign for a specific point based on the given graph and locating the *x*-intercept of the derivative graph from its corresponding original function. An excerpt from the interview with S9 is as follows:

Instructor : Where is the x-intercept for the derivative function?
S9 : For quadratic graph…um**...** I can't remember it… like how?

(Interview S9: 347-353)

S7 and S8 encountered challenges in assessing the value sign of $f''(x)$ and resorted to conjecture due to the absence of the relevant graph. The identified difficulties suggested a requirement for additional instruction and practice in graph interpretation, key point identification, and comprehending the correlation between the original function and its derivative.

Misconceptions on the Derivative Concepts. The analysis identified student misconceptions regarding differentiation concepts. One considerable example was S8 misinterpreting the derivative of a cubic function as linear but later recognising it as quadratic upon understanding the highest power of the x variable in a cubic graph. This observation underscored students' challenges in interpreting graphical representations without the corresponding equations. Consequently, these points highlighted the necessity of improving instruction on graphical representations and their connection with mathematical concepts. Excerpts from the interview with S8 are as follows:

Instructor : In this case, since we have a cubic graph, what kind of format do you think the derivative graph will take? Will it be linear, quadratic, or cubic?
S8 : It will be linear.

(Interview S8: 58-59)

Instructor : What are the differences between the derivative graphs?
S8 : From like cubic to u-shaped or n-shaped.
Instructor : Oh, okay. So, I mean to say, is your derivative function now u-shaped or n-shaped?
S8 : Yup.

(Interview S8: 80-83)

S7 erroneously concentrated exclusively on the increasing or decreasing lines when assessing the value sign at a specific point, overlooking the position of the point about the *x*-axis. This outcome suggested a misconception on examining points on a graph thoroughly. Thus, highlighting the significance of the location of the point and the behaviour of the graph could address this misconception while enhancing the understanding of derivative concepts.

Interview Outcomes for Average Category Students

Understanding of Derivative Concepts. All three students (S4, S5, and S6) demonstrated a commendable understanding of derivative concepts associated with curve properties. The participants understood instructions for the questions, exhibiting proficiency in applying their knowledge. S4 and S5 achieved the Action stage, while S4 reached the Process stage. A superior understanding was also observed for S4 by accurately identifying the value sign for $f''(x)$ from the $f'(x)$ graph, reflecting a more profound grasp of relationships between graphs and their derivatives.

All students recognised the derivative graph shape of cubic functions for its quadratic form. Nevertheless, S4 and S5 noted that the functions could resemble either an n-shaped or u-shaped, indicating a more nuanced understanding. S5 also acknowledged that the x-intercepts of the derivative graph corresponded to the stationary points of the original function. Additionally, the comprehension of S5 on positive and negative values for dy/dx based on function behaviour indicated a thorough grasp of derivative concepts.

Task Difficulty. Although all students comprehended the relationship with stationary points, S4 struggled to identify the x-intercept on the derivative graph during the interview when dy/dx equalled zero. This finding highlighted the need to clarify the correlation between zero derivative values and x-intercepts. An excerpt from the interview with S4 is as follows:

Instructor	: So, what happens to the derivative for both graphs?
S4	: Well, they both intersect at x = -1.
Instructor	: Correct. Now, what do we call that point?
S4	: What do we call it? Um, x = -1?
Instructor	: Yes, but what about the y-coordinate?
S4	: The y-coordinate is different for both graphs.

(Interview S4: 123-128)

Misconceptions on the Derivative Concepts. S6 accurately understood that the derivative of cubic functions was quadratic. Nonetheless, this student mistakenly linked u-shaped to the derivative of quadratic functions. This result demonstrated a minor misunderstanding regarding the characteristics of quadratic and cubic derivatives. S6 also presented a common misconception by concentrating on the increasing or decreasing line to ascertain the value sign at a specific point while overlooking the essential position factor of the point relative to the x-axis. This misconception could cause erroneous property interpretations of the function and its derivatives. An excerpt from the interview with S6 is as follows:

Instructor	: Let's discuss the value sign for $f'(x)$. Can you explain why the signs are positive and negative in this section?
S6	: Okay, because this is the first derivative graph.
S6	: When $x = a$, the graph reaches its maximum point and increases.
Instructor	: Oh, then for $x = b$?
S6	: When $x = b$, the graph starts to decrease, so I believe that at $x = b$, the sign should be negative.

(Interview S6: 32-33; 37-39)

Interview Outcomes for Above-Average Category Students

Understanding of Derivative Concepts. The students in the above-average category exhibited a robust foundational understanding, applying derivative concepts effectively to reach the Action stage. These participants clearly understood the instructions for the questions and articulated them effectively. Moreover, these students accurately determined the value signs based on point location, showcasing proficiency in graph analysis. The students also recognised the alignment of derivative graph x-

intercepts with the stationary points of the original function, indicating a comprehensive understanding of graph relationships. Specifically, S1 demonstrated a more profound knowledge by linking decreasing line segments with negative dy/dx values and vice versa. Consequently, the comprehension of sign changes in the derivative enhanced the interpretation of the graphical representation and the behaviour of the function.

Task Difficulty. S2 encountered a challenge in identifying the value sign of the second derivative from the first derivative graph in the absence of the equation for the original function. This observation implied a necessity for additional direction in linking the visual representation of the graph to its fundamental mathematical characteristics. An excerpt from the interview with S2 is as follows:

Instructor : Then how about the value sign $f''(x)$?
S2 : Hmm... I think it will be the opposite. Maybe one part will go down, and the other will go up. I wasn't sure as the equation of $f'(x)$ was not provided.

(Interview S2: 45-46)

Misconceptions on the Derivative Concepts. S3 demonstrated a misconception by concentrating on the trend of the line when assessing the value sign for the original graph, which did not align with the intended question. The method employed was incorrect. Although the technique was appropriate to ascertain the value sign of the derivative graph from the original graph, it was not the correct method for determining the value sign of the provided original graph. This misconception denoted the necessity of clearly distinguishing between the analysis of the original function and its derivative. An excerpt from the interview with S3 is as follows:

Instructor : Can you explain how to get the value sign for this graph?
S3 : Because it is increasing, um… and then… this one is decreasing.

(Interview S3: 108-109)

DISCUSSION

The task-based interview outcomes of this study, with students of differing performance levels on derivative concepts, presented quality data on their understanding. Given that the outcomes facilitated future advancements in the instruction of derivative concepts, several disciplines could benefit from this information for further exploration and integration into instructional strategies.

This study observed Action-stage engagement for students' performance in the below-average category on derivative tasks. Ng and Chiew (2023) presented similar outcomes, in which conceptual errors characterised low achievers. Most students then demonstrated a Process-stage understanding during the interview, which was consistent with the results reported by Maharaj (2013), Nagle et al. (2019), and Maharaj and Ntuli (2018). Considering that Jones and Watson (2018) proposed multiple suggestions, the content of the DCIL instructional unit for this study could also be modified. One proposed example was the "target understanding" approach to the derivative concept. This method could enhance the teaching and learning of calculus, focusing on achieving a comprehensive grasp of derivatives through graphical, symbolic, verbal/rate, and physical contexts.

Similarly, Feudel and Biehler (2021) developed a theoretical model to define a meaningful understanding of the derivative, specifically within the context of economics. Their exploratory study revealed that many economics students did not acquire conceptual understanding by the end of their calculus courses. In particular, they struggled with interpreting derivatives in common economic contexts, indicating persistent gaps and misconceptions. This reinforces the broader concern that derivative concepts are often learned procedurally rather than conceptually and highlights the need for instructional designs that connect formal calculus concepts with meaningful applications in various domains.

This study highlighted that targeted interventions were necessary due to the challenges encountered by specific students. One significant example was the misconceptions about zero values of derivatives and their relationship to *x*-intercepts. Thus, interventions such as explicit instruction and illustrative examples within the Desmos platform for visualisation, could address this issue. Various unique characteristics of quadratic with cubic derivatives and the positive values in derivative functions could also resolve the misinterpretations of their distinction. García-García and Dolores-Flores (2021) emphasised that visualisation is fundamental to mathematical comprehension. McGee and Moore-Russo (2015) demonstrated effective representation in comprehension by employing 3D slopes as foundational knowledge before exploring 3D derivatives.

Considering the difference between algebraic and graphical, instruction and practice should be integrated. Integrating visual aids and graphing activities helps students connect symbolic procedures with graphical behaviour. Visual tools that emphasise key features such as inflection points and graph behaviour boost effectiveness in improving understanding and pattern recognition. Lastly, technological tools, particularly graphing software, could enhance students' exploration of graphical representations and deepen their understanding.

CONCLUSION

This study successfully conducted interview sessions, yielding quality data regarding students' strengths, challenges, and misconceptions across various performance categories in understanding derivative concepts and their applications to curve properties. An improved understanding of derivative concepts could be achieved through targeted instruction and modification of the instructional units. Nonetheless, future studies should examine the effectiveness of various instructional methods. One suggestion involves guided graph exploration and interactive activities to discern the optimal strategies for teaching curve properties related to derivative concepts. Consequently, educators can develop instructional strategies that address students' varied needs and learning styles.

The individual student interviews performed in this study produced excellent data for enhancing the teaching of derivative concepts. Therefore, students' understanding of derivatives should be improved by targeting specific areas of difficulty and misconception, utilising visual representations, and adapting instructional materials through optimal instructional strategies. Furthermore, ongoing research and development in teaching experiments can advance instructional strategies and improve student proficiency in derivatives.

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